

【課題 1-3】

■ 近似解の仮定 以下のように近似解を仮定します。

$$u(x) = ax, \quad \delta u(x) = bx, \quad \varepsilon = a, \quad \delta \varepsilon = b \quad (1-3.1)$$

■ 仮想仕事の原理の適用 仮想仕事の原理に対して式(1-3.5)を代入して,

$$\begin{aligned} \int_0^L \sigma \delta \varepsilon A dx &= E \delta u \Big|_{x=L} \rightarrow \int_0^L E a b A dx = F b x \Big|_{x=L} = F b L \\ \therefore b \left(E a \int_0^L A dx - F L \right) &= 0 \end{aligned} \quad (1-3.2)$$

式(1-3.2)が任意の b に対して恒等的に成り立つためには,

$$E a \int_0^L A dx = F L \quad \therefore a = \frac{F L}{E \int_0^L A dx} \quad (1-3.3)$$

式(1-3.3)の右辺の分母の積分を計算すると,

$$\begin{aligned} \int_0^L A dx &= \int_0^L \pi \left(\frac{r_2 - r_1}{L} x + r_1 \right)^2 dx \\ &= \frac{\pi L}{3(r_2 - r_1)} \left[\left(\frac{r_2 - r_1}{L} x + r_1 \right)^3 \right]_0^L = \frac{\pi L}{3(r_2 - r_1)} ((r_2 - r_1) + r_1)^3 \\ &= \frac{\pi L}{3(r_2 - r_1)} (((r_2 - r_1) + r_1)^3 - r_1^3) = \frac{\pi L}{3(r_2 - r_1)} (r_2^3 - r_1^3) \\ &= \frac{\pi L (r_2 - r_1)(r_2^2 + r_1 r_2 + r_1^2)}{3(r_2 - r_1)} = \frac{\pi L (r_2^2 + r_1 r_2 + r_1^2)}{3} \end{aligned} \quad (1-3.4)$$

式(1-3.4)を式(1-3.3)に代入して

$$\begin{aligned} a &= \frac{F L}{E \int_0^L A dx} = \frac{3 F L}{E \pi L (r_2^2 + r_1 r_2 + r_1^2)} = \frac{3 F}{E \pi (r_2^2 + r_1 r_2 + r_1^2)} \\ \therefore u(x) &= \frac{3 F x}{E \pi (r_2^2 + r_1 r_2 + r_1^2)} \end{aligned} \quad (1-3.5)$$

■ 近似解の評価 式(1-3.9)は $r = r_1 = r_2$ のとき,

$$u(x) = \frac{F x}{E \pi r^2} \Big|_{x=L} = \frac{F L}{E A}$$

【課題解答例】

となり、本文中の式(1-4)を満足します。【課題 1-1】の式(1-1.4)と式(1-3.5)の両辺に $E\pi r_1^2/F$ を乗じて

$$\frac{E\pi r_1^2 u(x)}{F} = \frac{x}{1 + \frac{(r_2 - r_1)x}{Lr_1}} \quad (1-3.6)$$

$$\frac{E\pi r_1^2 u(x)}{F} = \frac{3x}{1 + \frac{r_2^2}{r_1^2} + \frac{r_2}{r_1}} \quad (1-3.7)$$

式(1-3.6)と式(1-3.7)を別々のシートに以下のように式定義してみます。

解析解[式(1-3.6)]

	A	B	C	D	E
1	L	100	100	100	100
2	r1	10	10	10	10
3	r2	10	9	8	7
4	r2-r1	=B3-B2	=C3-C2	=D3-D2	=E3-E2
5	L*r1	=B2*B1	=C2*C1	=D2*D1	=E2*E1
6	x	Case1-1	Case1-2	Case1-3	Case1-4
7	0	=\$A7/(1+B\$4*\$A7/B\$5)	=\$A7/(1+C\$4*\$A7/C\$5)	=\$A7/(1+D\$4*\$A7/D\$5)	=\$A7/(1+E\$4*\$A7/E\$5)
8	1	=\$A8/(1+B\$4*\$A8/B\$5)	=\$A8/(1+C\$4*\$A8/C\$5)	=\$A8/(1+D\$4*\$A8/D\$5)	=\$A8/(1+E\$4*\$A8/E\$5)
9	2	=\$A9/(1+B\$4*\$A9/B\$5)	=\$A9/(1+C\$4*\$A9/C\$5)	=\$A9/(1+D\$4*\$A9/D\$5)	=\$A9/(1+E\$4*\$A9/E\$5)
10	3	=\$A10/(1+B\$4*\$A10/B\$5)	=\$A10/(1+C\$4*\$A10/C\$5)	=\$A10/(1+D\$4*\$A10/D\$5)	=\$A10/(1+E\$4*\$A10/E\$5)
11	4	=\$A11/(1+B\$4*\$A11/B\$5)	=\$A11/(1+C\$4*\$A11/C\$5)	=\$A11/(1+D\$4*\$A11/D\$5)	=\$A11/(1+E\$4*\$A11/E\$5)
12	5	=\$A12/(1+B\$4*\$A12/B\$5)	=\$A12/(1+C\$4*\$A12/C\$5)	=\$A12/(1+D\$4*\$A12/D\$5)	=\$A12/(1+E\$4*\$A12/E\$5)
13	6	=\$A13/(1+B\$4*\$A13/B\$5)	=\$A13/(1+C\$4*\$A13/C\$5)	=\$A13/(1+D\$4*\$A13/D\$5)	=\$A13/(1+E\$4*\$A13/E\$5)
14	7	=\$A14/(1+B\$4*\$A14/B\$5)	=\$A14/(1+C\$4*\$A14/C\$5)	=\$A14/(1+D\$4*\$A14/D\$5)	=\$A14/(1+E\$4*\$A14/E\$5)
15	8	=\$A15/(1+B\$4*\$A15/B\$5)	=\$A15/(1+C\$4*\$A15/C\$5)	=\$A15/(1+D\$4*\$A15/D\$5)	=\$A15/(1+E\$4*\$A15/E\$5)
16	9	=\$A16/(1+B\$4*\$A16/B\$5)	=\$A16/(1+C\$4*\$A16/C\$5)	=\$A16/(1+D\$4*\$A16/D\$5)	=\$A16/(1+E\$4*\$A16/E\$5)
17	10	=\$A17/(1+B\$4*\$A17/B\$5)	=\$A17/(1+C\$4*\$A17/C\$5)	=\$A17/(1+D\$4*\$A17/D\$5)	=\$A17/(1+E\$4*\$A17/E\$5)

近似解[式(1-3.7)]

	A	F	G	H	I
1	L	100	100	100	100
2	r1	10	10	10	10
3	r2	10	9	8	7
4	r2-r1	=F3-F2	=G3-G2	=H3-H2	=I3-I2
5	L*r1	=F2*F1	=G2*G1	=H2*H1	=I2*I1
6	x	Case2-1	Case2-2	Case2-3	Case2-4
7	0	=3*\$A7/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A7/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A7/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A7/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
8	1	=3*\$A8/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A8/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A8/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A8/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
9	2	=3*\$A9/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A9/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A9/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A9/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
10	3	=3*\$A10/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A10/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A10/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A10/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
11	4	=3*\$A11/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A11/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A11/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A11/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
12	5	=3*\$A12/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A12/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A12/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A12/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
13	6	=3*\$A13/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A13/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A13/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A13/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
14	7	=3*\$A14/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A14/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A14/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A14/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
15	8	=3*\$A15/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A15/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A15/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A15/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
16	9	=3*\$A16/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A16/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A16/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A16/(1+(I\$3/I\$2)^2+(I\$3/I\$2))
17	10	=3*\$A17/(1+(F\$3/F\$2)^2+(F\$3/F\$2))	=3*\$A17/(1+(G\$3/G\$2)^2+(G\$3/G\$2))	=3*\$A17/(1+(H\$3/H\$2)^2+(H\$3/H\$2))	=3*\$A17/(1+(I\$3/I\$2)^2+(I\$3/I\$2))

グラフ化すると、以下のように $r_1 = r_2$ のとき正確な値となり、これらの値

が離れると誤差が増えてきます（実線が解析解，点線が近似解）。

