【課題 1-1】

まず，　$\varepsilon = \dfrac{\sigma}{E}$，　$u(x) = \int \dfrac{du}{dx} dx = \int \varepsilon \, dx = \int \dfrac{\sigma}{E} dx$　　(1-1.1)

ですから，

$$u(x) = \int \frac{F}{\pi E} \left(\frac{r_2 - r_1}{L} x + r_1 \right)^{-2} dx = -\frac{FL}{\pi E (r_2 - r_1)} \left\{ \left(\frac{r_2 - r_1}{L} x + r_1 \right)^{-1} + C \right\} \quad (1\text{-}1.2)$$

となります。境界条件 $u(0) = 0$ から，$C = -1/r$ すなわち

$$u(x) = -\frac{FL}{\pi E (r_2 - r_1)} \left\{ \left(\frac{r_2 - r_1}{L} x + r_1 \right)^{-1} - \frac{1}{r} \right\}$$

$$= -\frac{FL}{\pi E (r_2 - r_1)} \left(\frac{L r_1 - (r_2 - r_1)x - L r_1}{r_1 ((r_2 - r_1)x + L r_1)} \right)$$

$$\therefore u(x) = \frac{FLx}{\pi E r_1 ((r_2 - r_1)x + L r_1)} \quad (1\text{-}1.3)$$

または，$u(x) = \dfrac{Fx}{E\pi r_1^2 \left(1 + \dfrac{(r_2 - r_1)x}{L r_1} \right)}$　　(1-1.4)

ここで，$r = r_1 = r_2$ のとき，$A = \pi r^2$ ですから

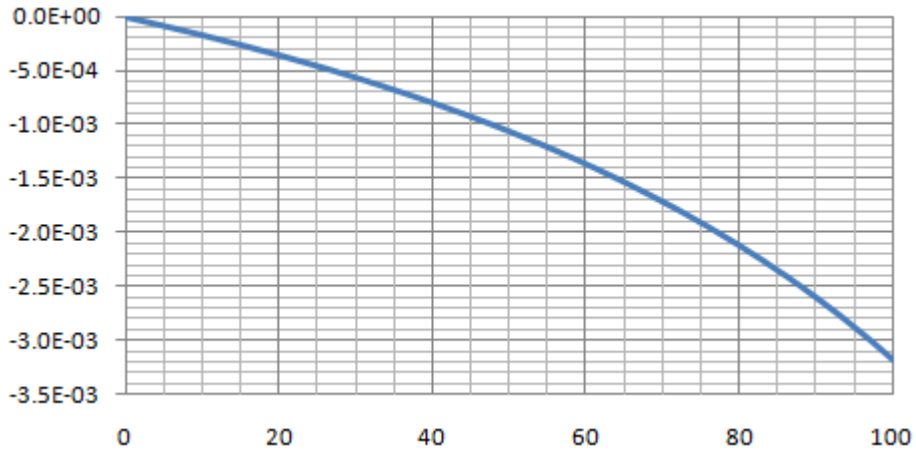
$$u(x) = \left. \frac{Fx}{E\pi r^2} \right|_{x=L} = \frac{FL}{EA}$$

となり，本文中の式(1-4)を満足しています。

■Excel シートの定義

	A	B	C	D	E	F	G
1	E=	500000	x	① Fx	② 1+(r_2−r_1)x/Lr_1	③Eπ r_1^2×②	④u(x)=①/③
2	F=	−10000	0	=$B$2*C2	=1+$B$9*C2	=$B$10*E2	=D2/F2
3	L=	100	1	=$B$2*C3	=1+$B$9*C3	=$B$10*E3	=D3/F3
4	r_1=	20	2	=$B$2*C4	=1+$B$9*C4	=$B$10*E4	=D4/F4
5	r_2=	10	3	=$B$2*C5	=1+$B$9*C5	=$B$10*E5	=D5/F5
6	r_2−r_1=	=B5−B4	4	=$B$2*C6	=1+$B$9*C6	=$B$10*E6	=D6/F6
7	r_1^2=	=B4^2	5	=$B$2*C7	=1+$B$9*C7	=$B$10*E7	=D7/F7
8	Lr_1=	=B3*B4	6	=$B$2*C8	=1+$B$9*C8	=$B$10*E8	=D8/F8
9	(r_2−r_1)/Lr_1=	=B6/B8	7	=$B$2*C9	=1+$B$9*C9	=$B$10*E9	=D9/F9
10	Eπ r_1^2=	=B1*PI()*B7	8	=$B$2*C10	=1+$B$9*C10	=$B$10*E10	=D10/F10
11			9	=$B$2*C11	=1+$B$9*C11	=$B$10*E11	=D11/F11
12			10	=$B$2*C12	=1+$B$9*C12	=$B$10*E12	=D12/F12
13			11	=$B$2*C13	=1+$B$9*C13	=$B$10*E13	=D13/F13
14			12	=$B$2*C14	=1+$B$9*C14	=$B$10*E14	=D14/F14

■変位の様子（散布図）


0.0E+00
-5.0E-04
-1.0E-03
-1.5E-03
-2.0E-03
-2.5E-03
-3.0E-03
-3.5E-03
0
20
40
60
80
100